

Directed Design of Experiments (DOE) for Determining Probability of Detection (POD) Capability of NDE Systems (DOEPOD)

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OUTLINE

- Binomial Distribution
- Process for determining observed probability of hit (POH) and associated confidence limits
- DOEPOD Software
- DOEPOD Analysis
- Future work



Background

"I'm too much of a knucklehead to know this stuff"

"Oh boy, confidence limits. I hate these."

"They don't use 90/50 they use 90/95."

"I should have used 90/50"

"I defer my answer to the statistician"

"I'm not a statistician."

"90/50 POD means that there is a 50% chance that the true POD is greater than 90% at that flaw size?" Responses: "No.", and "Yes.", rest of world gives blank stares.

"Confusion over 'common definitions' continues to be an issue..."

"We have been using 29 out of 29 clandestinely for years"



Background (continued)

- A core issue here is that the NDE personnel, nationwide, have different levels of understanding of statistics, and have delegated basic NDE POD statistical analysis to the statisticians.
- This environment created a divergence in the interrelationship between the physics of the inspection procedure and the POD statistics.
- NDE community should not blindly accept statistical results, but rather challenge the statistical results.
- When NDE personnel defer explanations on statistical confidence bounds to others, it's like saying "I don't understand the error bars of my data".
- This is not a good position.
- We all need to learn and to speak the language of the other.
- This is the authors attempt to begin to bridge this gap.

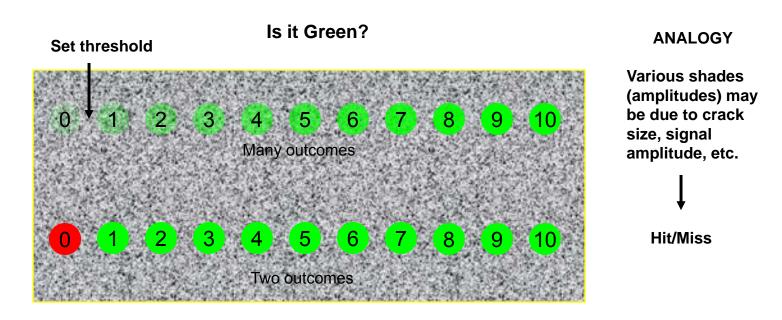


Using The Binomial Distribution

The binomial distribution describes the behavior of a count variable X if the following conditions apply:

- > The number of observations N is fixed.
- > Each observation is independent.
- > Each observation represents one of two outcomes ("success" or "failure").
- ➤ Use "green" or "red" to represent "Hit" or "Miss", respectively.
- > The probability of "Hit" (POH) is the same for each outcome.

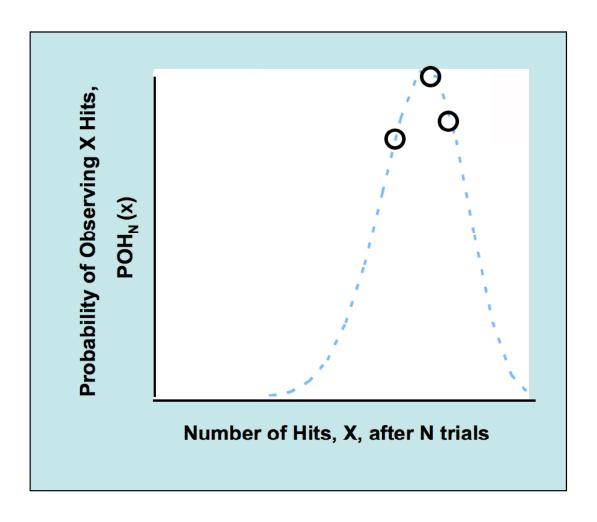
If these conditions are met, then X has a binomial distribution.



By setting a threshold only two outcomes (Hit/Miss) observations are obtained.



Using The Binomial Distribution (continued)



O = Example values of POH as observed during a test.

Use binomial distribution for now.

Other distributions may be used if they can be demonstrated to be better.



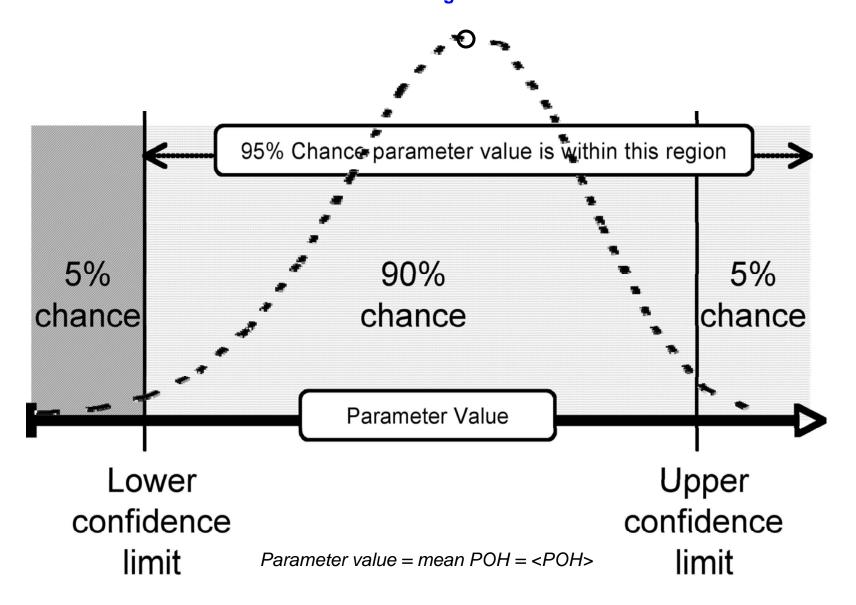
Probability of Hit (POH) Example

- Start with 61 flaws in the group.
- Each flaw has with the same probability of being observed as a Hit.
- Make 61 observations.
- If 59 Hits are observed, then the Probability of Hit is POH = 59/61 = 0.97 (the observed frequency)
- This is an estimated POH since the true POH can only be approached by making an "infinite" amount of observations.
- Now that the Probability of Hit is measured, what is the confidence in that value?
- This is somewhat analogous to asking what are the error bars or uncertainty in measurements.
- Confidence level is the measure of probability associated with a confidence interval expressing the probability of truth of a statement that the interval will include the parameter value.
- For NDE applications, the confidence bound of interest for Probability of Hit is the lower confidence bound.



95% Confidence Level Example

If we achieve a lower confidence limit (bound, value) = 0.90, then
"There is a 95% chance that the true POH is greater than 90% for that flaw size"





Confidence Level Example (continued)

With X = 59 Hits after N = 61 trials, yielding 59/61 = 0.97 POH (the observed frequency), the lower confidence bound, P_{ij} , may be obtained from*

$$P_{l} = \frac{X}{X + (N - X + 1)F_{\alpha}(f_{1}, f_{2})}, F_{\alpha}(f_{1}, f_{2}) = 2.25 \begin{cases} f_{1} = 2(N - X + 1) = 6 \\ f_{2} = 2X = 118 \end{cases}$$

$$P_1 = 0.9$$

 α is, a priori, the confidence level, 95%, that we are requiring $F_{\alpha}(f_1,f_2)$ is obtained from the F-distribution statistical table

Note that the POH does NOT change if the confidence level is changed



95% Confidence Level Example (continued)

$$P_l = 0.9$$

There is an 95% chance that the true POH is greater than 0.9 at that flaw size

Or

There is a 95% chance that the inspection system reliability is greater than 0.9 at that flaw size

Or

90/95 POD at that flaw size



Directed Design of Experiments for Validating Probability of Detection of Inspection Systems (DOEPOD)

DOEPOD Concepts

- ➤ Not all flaws are created equally; never identical; but they may be grouped into classes by size, length, depth, etc. These classes have ranges or widths.
- ➤ DOEPOD is a confidence value driven approach.
- ➤ DOEPOD uses moving class width and variable class width optimization to identify the best lower confidence bound. Class widths start at 0.001" and increase.
- ➤ DOEPOD uses real initial flaw, simulated, or completed inspection data sets. Guidelines are in the manual. (Smallest number of samples is 5).

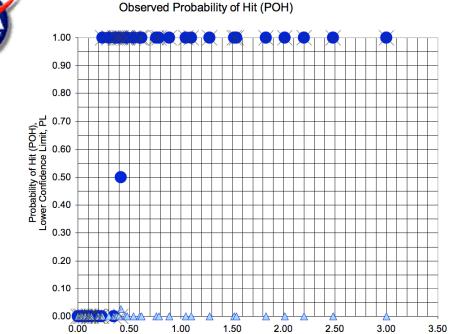


Grouping of Flawed Specimens

- Flaws may be grouped into classes by size, length, depth, etc. The grouping may be any class width, e.g., 0.001", 0.036", 0.100", etc.
- If there are sufficient number of flaws, then a moving "class width" may be used to dynamically group adjacent flaws into classes with widths (classwidths).
 - ➤ E.g., all flaws in the range 0.050" 0.150" may be in a group, with the largest flaw being the identifier for the group.
 - Class width here is 0.100"
 - The next group may contain the range 0.049" 0.149"; the class width is moving from largest to smallest flaws.
- POH needs to be determined for each flaw size grouping (number of flaws in each group is not necessarily the same)
- Confidence bounds need to be determined for each flaw size grouping (both X and N vary for each group)

What is the optimum class width?





Size (inch), X1

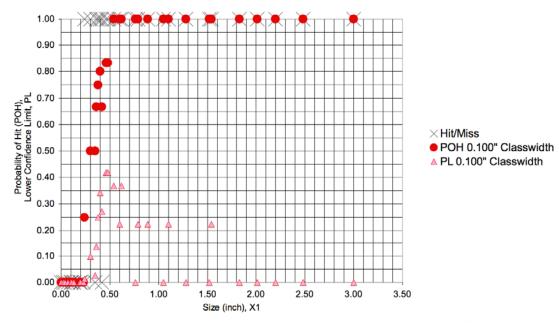
Effect of Grouping of Flawed Specimens

● POH 0.001" Classwidth △ PL 0.001" Classwidth

×Hit/Miss

Observed Probability of Hit (POH)

Important: When grouping flaws, the observed POH and confidence bound is assigned to the largest flaw in the group





DOEPOD

DOEPOD Concepts (continued)

- Using POH lower confidence bound (value) as driver for directing DOE.
- ➤ If 90/95 POD is reached at a then identify locations that need additional vaildation for larger flaw sizes.
- ➤ If 90/95 POD is not reached then use best lower confidence value to identify where options are available to reach 90/95 POD.
- ➤ Identify CASE of the data set.
- ➤ Provide directions, depending on the CASE, on how to modify the DOE to continue to efficiently validate the inspection system.
- > Determine false call rate and associated confidence limit



DOEPOD Parameters

Class length Inspection parameter (length, depth, etc.)

Hit Flaw is detected

Miss Flaw is not detected

Need Add new samples to the existing set in order to reach the number of samples

required at the class length

LCL Lower confidence bound (value) of POH @ 95% confidence

Opt. X_{POH} Optimum X_{POH}

Optimum X_{POH} is identified for non-survey data sets. Optimum X_{POH} is the smallest class length and largest class width at which the minimum $X_{POH} = 1$ occurs. Optimum X_{POH} may be more aggressive than optional, X_{PODopt}, or X_{Best LCL}, when the class width is constrained to the companion Optimum X_{POH} class width listed. DOEPOD does not force use of Optimum X_{POH} over X_{PODopt.} or X_{Best LCL} Stability has not been demonstrated at Optimum X_{POH}, therefore there is an additional risk that can not be satisfied.

POH Probability of Hit (Number of Hits in Classwidth/Total Number of Trials in Classwidth)

Probability of Detection (the true POD obtained if an infinite number of samples are used) POD

Signal **Amplitude**

Sets

Scalar amplitude output of NDE inspection system

Survey Data

Survey Data Sets are data sets that have a sparse or disperse collection of samples. The moving class width optimization has identified this set as having limited applications to moving class width processing. An alternate optimization of X_{POH} is used to provide guidance.

Survey X_{POH} Survey X_{POH} is only identified for data sets determined to be Survey Data Sets. Survey X_{POH} is the smallest class length and largest class width at which the minimum $X_{POH} = 1$ class length occurs.

> Survey X_{POH} is the minimum class length at which X_{POD} may be achieved when the class width is constrained to the companion survey class width listed. Survey X_{POH}is utilized in all cases in which it occurs.



DOEPOD Parameters (continued)

X_{Best LCL} Class length exhibiting the best LCL. The best class length is

determined by increasing the moving class width until a

maximum LCL is obtained

X_i Class length X at point "i"

X₁ Largest class length in entire data set

X_m Class length near the mid-point between the largest and the smallest

class lengths having no misses

X_{POD} Class length at which the lower confidence bound (value) is 0.90 or

greater (90/95 POD)

 $X_{POH=1}$ Class length where there are no misses above this class length

 X_{PODopt} Recommended optional existing smaller class length where X_{POD} may

also be achieved if additional sample are added.

X_s Smallest class length in data set

UCL Upper confidence value of the false call rate @ 95% confidence



Case #1 (Best Case)

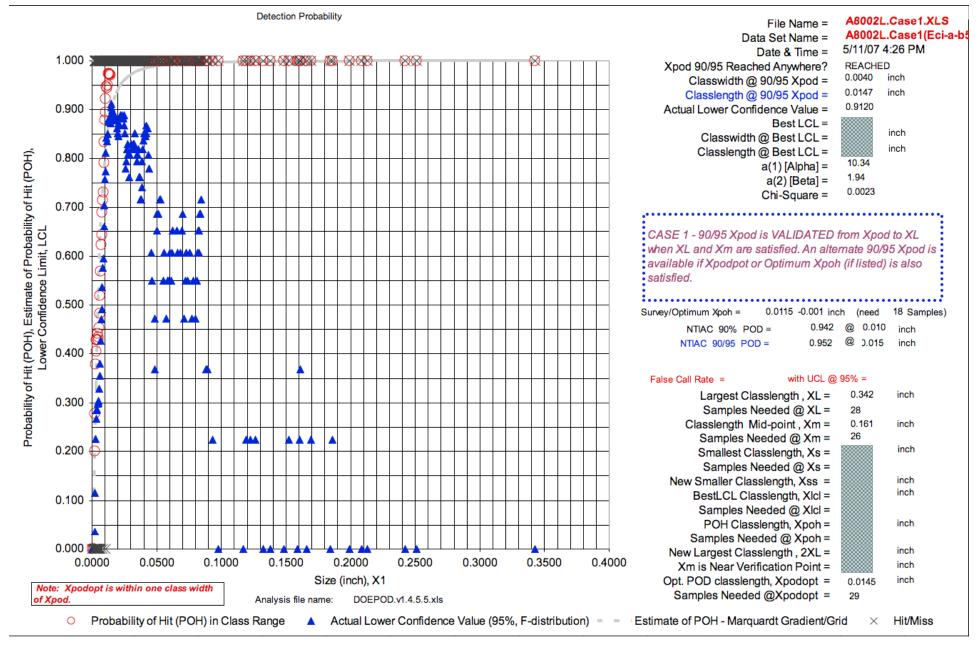
- 90/95 X_{POD} reached at a class length
- Misses only below X_{POD} (and POH =1 everywhere greater than X_{POD})

Directed Requirements for Validation of 90/95 X_{POD}

- Need samples at largest class length, X_L
- ➤ Need samples at mid-class length, $X_m \approx (X_L X_{POD})/2$
- \triangleright Option: Adding samples at X_{podopt} may yield a new X_{POD} with a smaller class length.
- \triangleright If this is a survey data set, then only need to add samples at Survey X_{POH} (if listed)
- \triangleright Option: The user may add samples at Optimum $X_{POH.}$ The class width for all added samples at any class length is shown along with the Optimum $X_{POH.}$
- \succ The range of validation may be expanded by adding samples at $2 X_L$, $4 X_L$, $8 X_L$, $16 X_L$, etc., if the current range of validation is too small.



Case #1 (Best Case)





- 90/95 X_{POD} reached at a class length
- There are misses below X_{POD} and some misses above X_{POD} . This is expected as X_{POD} nears the capability of the inspection system.
- Since misses exist at class lengths X_i above X_{POD} , then these greater lengths need to be validated. (i.e., The POH < 1 at class lengths X_i above X_{POD} point, X_{POD} , so these greater lengths need to be validated.)

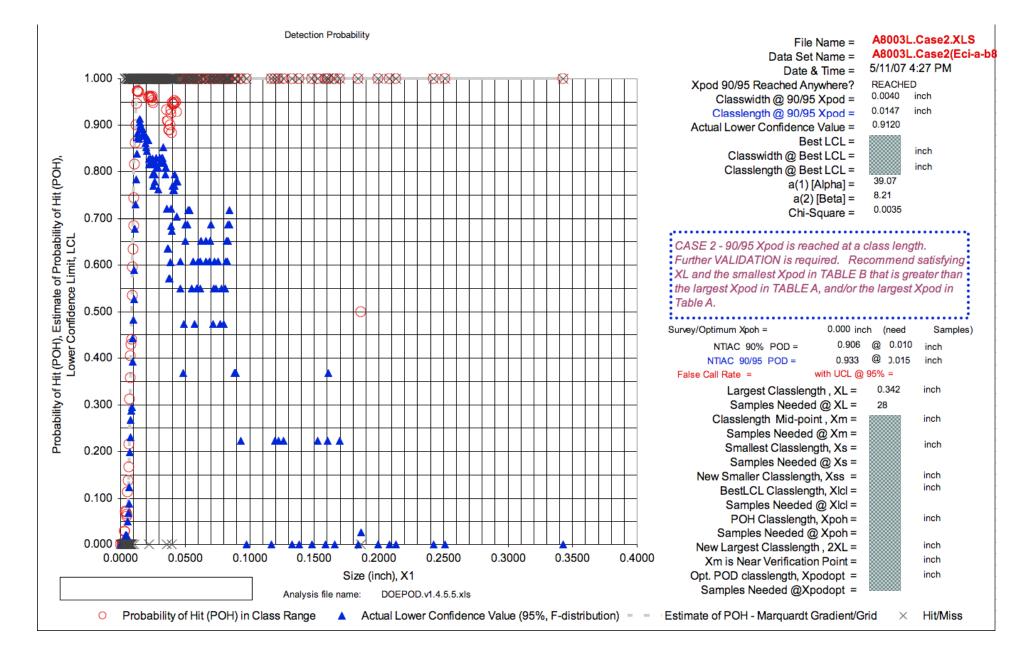
Directed Requirements

- ➤ There are two (2) options that may be used to move this Case #2 toward Case #1
 - (a) Add samples of class length X_i where POH<1 (TABLE A). Starting from largest class length, X_i and work toward small class lengths until reaching an acceptable X_{POD} or reaching X_{POD}
 - (b) Add samples of class length X_i where POH=1 (TABLE B). Accept a larger X_{POD} class length at any of the X_i . This acceptance is valid as long as any existing larger class lengths where POH<1 are shown [via (a) above] to be at 90/95 X_{POD} or greater. Acceptance of a larger X_{POD} is not necessarily the X_{POD} capability for the inspection system.

In summary, satisfy the smallest X_{POD} in Table B that is greater than the largest X_{POD} in Table A, and/or the largest X_{POD} in Table A.

- \triangleright If this is a survey data set, then only need samples at Survey X_{POH} (if listed), rather than at $X_{POH=1}$
- \triangleright Option: If Optimum $X_{POH} < X_{POH=1}$ then the user may add samples at Optimum X_{POH} rather than at $X_{POH=1}$. The class width for all added samples at any class length is shown along with the Optimum X_{POH} .
- Need samples at largest class length, X_L

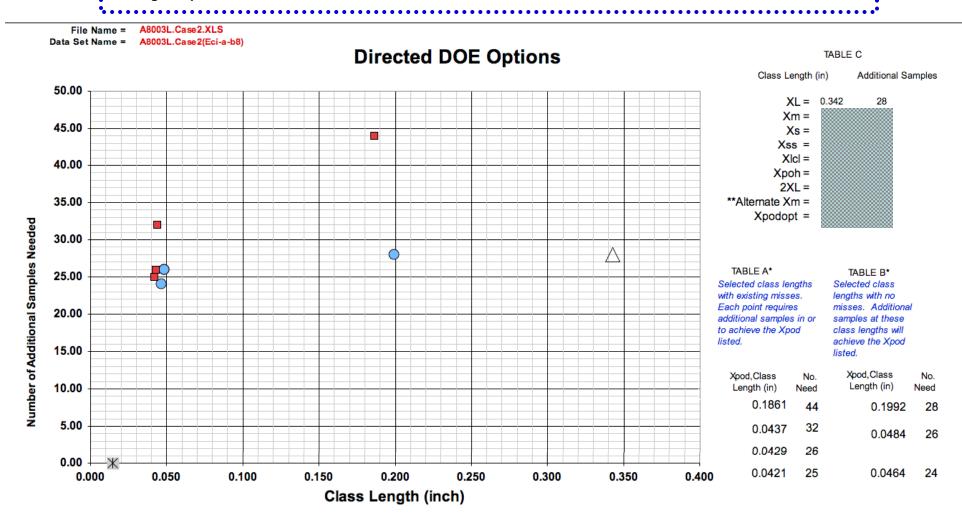




CASE 2 - 90/95 Xpod is reached at a class length.

Further VALIDATION is required.

Recommend satisfying XL and the smallest Xpod in TABLE B that is greater than the largest Xpod in TABLE A, and/or the largest Xpod in Table A.





- 90/95 X_{POD} Not Reached
- Best LCL is below 0.9 for the best class width group
- No misses at or greater than class length exhibiting the best LCL, $X_{Best\ LCL}$ (i.e., POH =1 everywhere at or greater than class length currently having the best LCL, $X_{Best\ LCL}$)

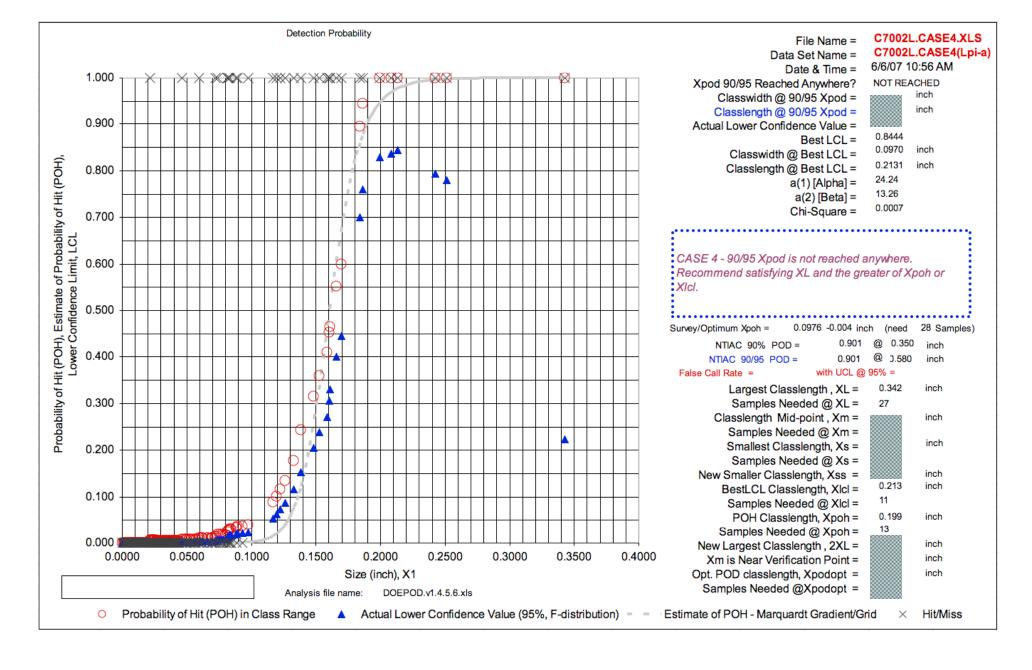
Directed Requirements

Need samples of $X_{Best\ LCL}$ in class length to achieve 90/95 X_{POD} at $X_{Best\ LCL}$. $X_{Best\ LCL}$ may equal X_{L} or $X_{POH=1}$ so that the number of samples listed at this class length are redundantly the same and only one set of samples is needed.

 \triangleright If this is a survey data set, then need to add samples at Survey X_{POH} (if listed), rather than at $X_{POH=1}$

 \triangleright Option: If Optimum $X_{POH} < X_{POH=1}$ then the user may add samples at Optimum X_{POH} rather than at $X_{POH=1}$. The class width for all added samples at any class length is shown along with the Optimum X_{POH}





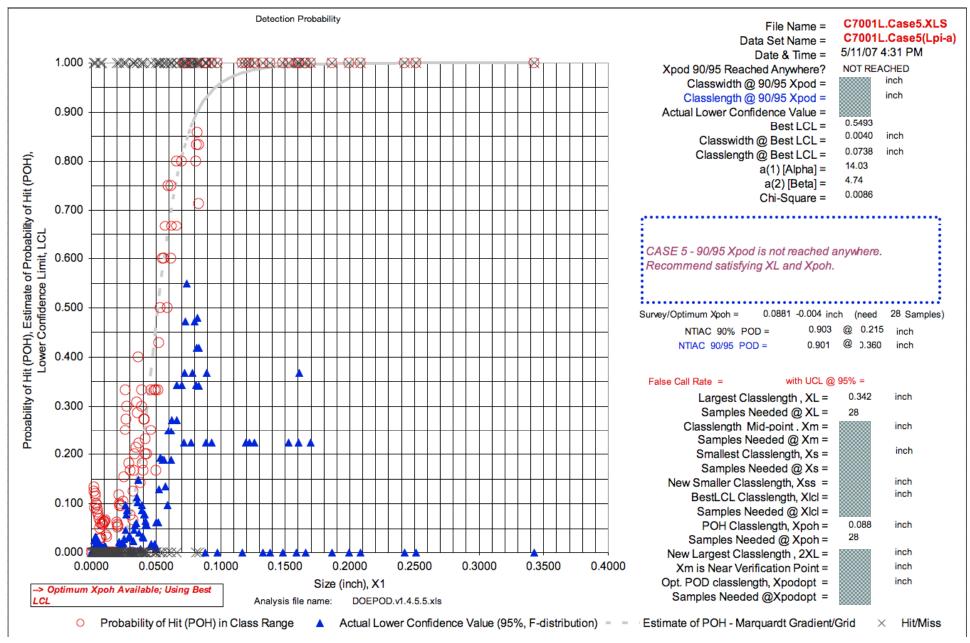


- 90/95 X_{POD} Not Reached
- Best LCL is below 0.9 for the best class width group
- There are misses at or greater than class length X_{Best LCL}
- There exists a class length, $X_{POH=1}$, above which there are no misses.
- There are no misses for class lengths equal to greater than $X_L/3$ (i.e., $X_{POH=1} \le X_L/3$)
- $X_{POH=1} \le X_L/3$ so that POH is not fluctuating at larger class lengths. Use $X_{POH=1}$ as the trial X_{POD}

Directed Requirements

- ➤ Need samples of X_{POH=1}
- \succ If this is a survey data set, then need to add samples at Survey X_{POH} (if listed), rather than at $X_{POH=1}$
- \triangleright Option: If Optimum $X_{POH} < X_{POH=1}$ then the user may add samples at Optimum X_{POH} rather than at $X_{POH=1}$. The class width for all added samples at any class length is shown along with the Optimum X_{POH} .
- Need 29 samples largest class length, X_L





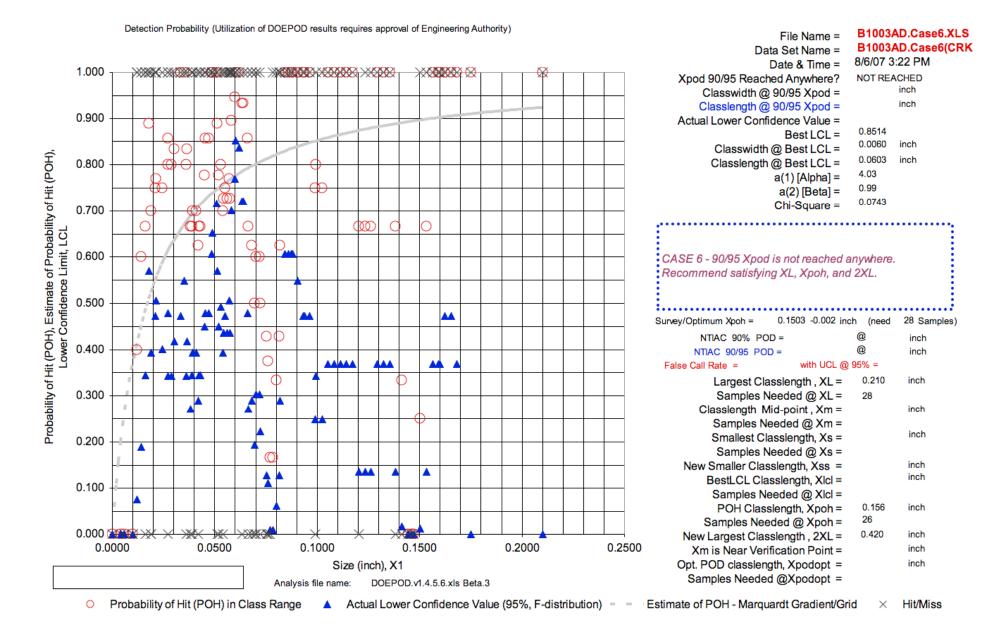


- 90/95 X_{POD} Not Reached
- Best LCL is below 0.9 for the best class width group
- There are misses at or greater than class length X_{Best LCL}
- There exists a class length, $X_{POH=1}$, above which there are no misses.
- There are misses for class lengths greater than $X_L/3$ (i.e., $X_{POH=1} \ge X_L/3$)
- $X_{POH=1} \ge X_L/3$ so that POH may be fluctuating rapidly.

Directed Requirements

- ➤ Need to expand current range of X₁ by adding new samples with class lengths of 2X₁ or greater
- \triangleright Need samples of $X_{POH=1}$. $X_{POH=1}$ may equal X_L so that the number of samples at this class length are redundantly the same and only one set of samples is needed.
- Need 29 samples at largest class length, 2X_L
- \triangleright If this is a survey data set, then need to add samples at Survey X_{POH} (if listed), rather than at $X_{POH=1}$
- \triangleright Option: If Optimum $X_{POH} < X_{POH=1}$ then the user may add samples at Optimum X_{POH} rather than at $X_{POH=1}$. The class width for all added samples at any class length is shown along with the Optimum X_{POH} .





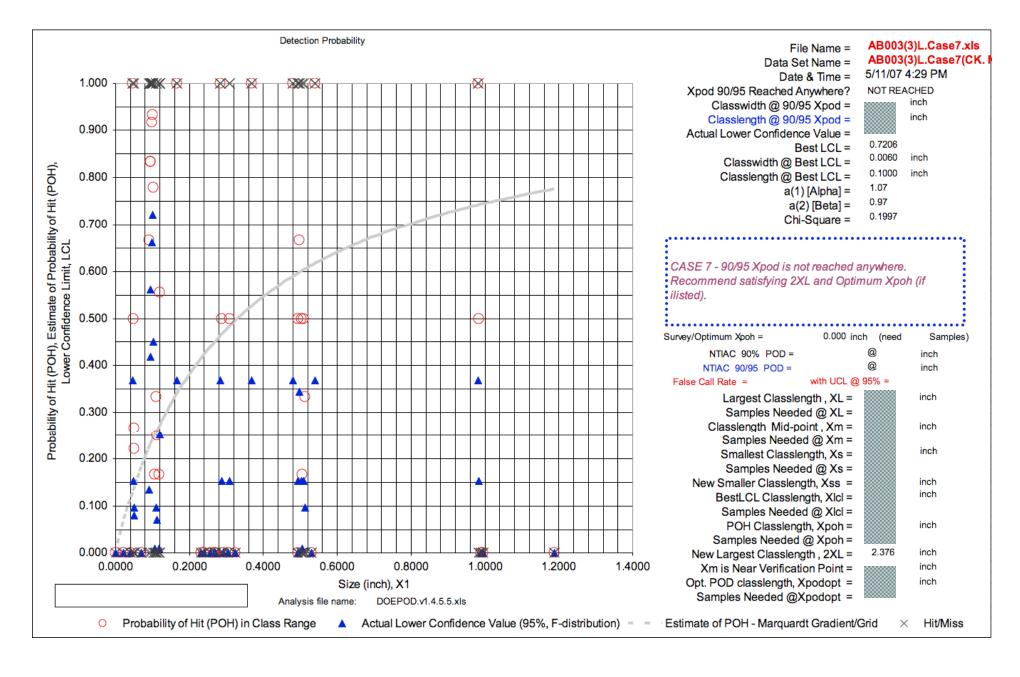


- 90/95 X_{POD} Not Reached
- Best LCL is below 0.9 for the best class width group
- There are misses at or greater than class length X_{Best LCL}
- There does **NOT** exist a class length, X_{POH=1}, above which there are no misses.
- POH may be fluctuating rapidly
- There may be no hits anywhere

Directed Requirements

- Inspection system may not be appropriate for meeting inspection criteria
- ➤ If this is a survey data set, then need to add samples at Survey X_{POH} (if listed)
- \triangleright Option: The user may add samples at Optimum X_{POH} . The class width for all added samples at any class length is shown along with the Optimum X_{POH} .
- ➤ Need to expand current range of X₁ by adding new samples with class lengths of 2X₁ or greater
- Need 29 samples at largest class length, 2X_L







Survey and Optimized X_{POH} Data Sets

- This data set has insufficient number of samples for unconstrained class width optimization
- The class width optimization has determined that there is a class width for which the smallest $X_{POH=1}$ class length is identified. The Survey and Optimum X_{POH} class lengths and class widths are identified on the charts as Survey/Optimum X_{POH} .
 - > For example, the listing:

Survey/Optimum $X_{POH} = 0.0500 - 0.015$ inch (need 18 samples)

indicates that a class width of 0.015" has been used and the Survey or Optimum X_{POH} occurs at 0.0500", and that 18 additional samples may be added to achieve X_{POD} add at that class length. The added samples should have sizes that range anywhere between 0.0500" and 0.035", inclusively.

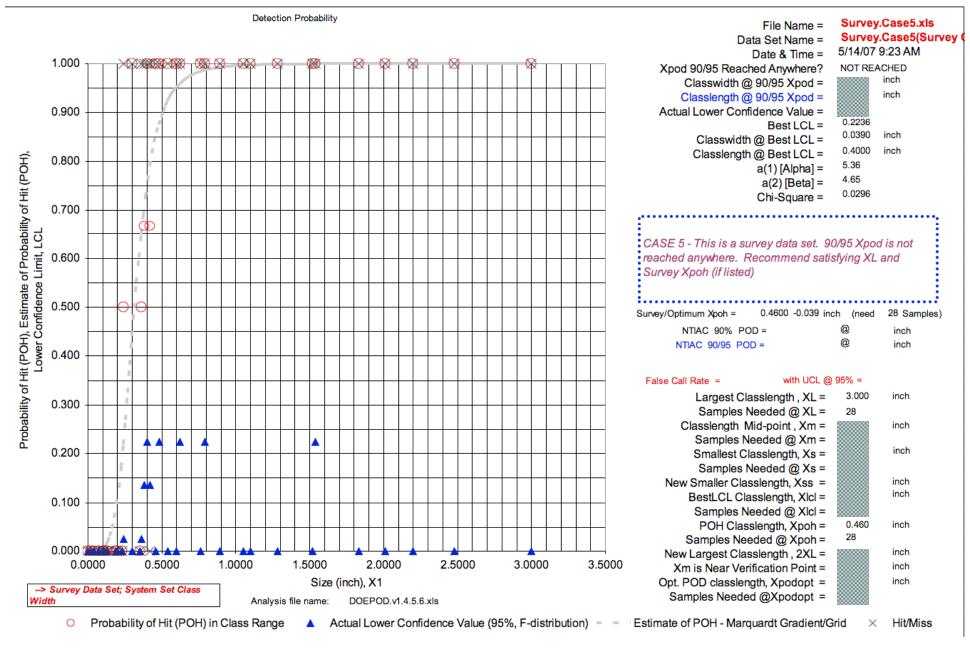
• If Survey/Optimum X_{POH} is pursued, then the class width for all added samples at any other class length is to be the same as that for Survey/Optimum X_{POH} .

Directed Requirements

- Need samples at Survey/Optimum X_{POH}
- ➤ Need samples at X₁
- Survey results are not available when user sets the class width



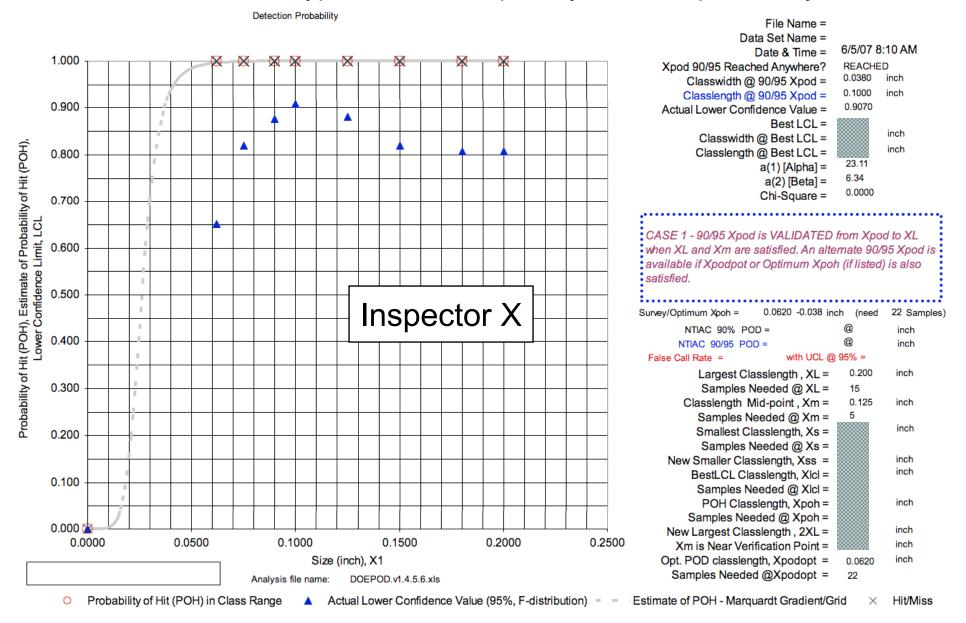
Case #5 - Survey





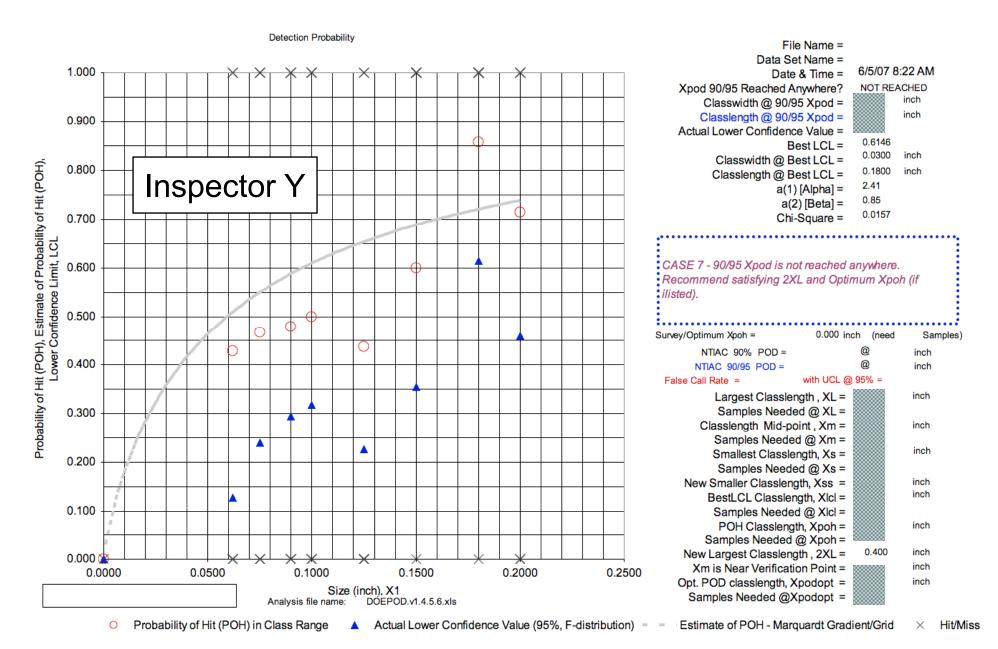
DOEPOD Example

Does the fastener type affect the capability of the inspection system?





DOEPOD Example (continued)



DOEPOD Example (continued)

Does the fastener type affect the capability of the inspection system?

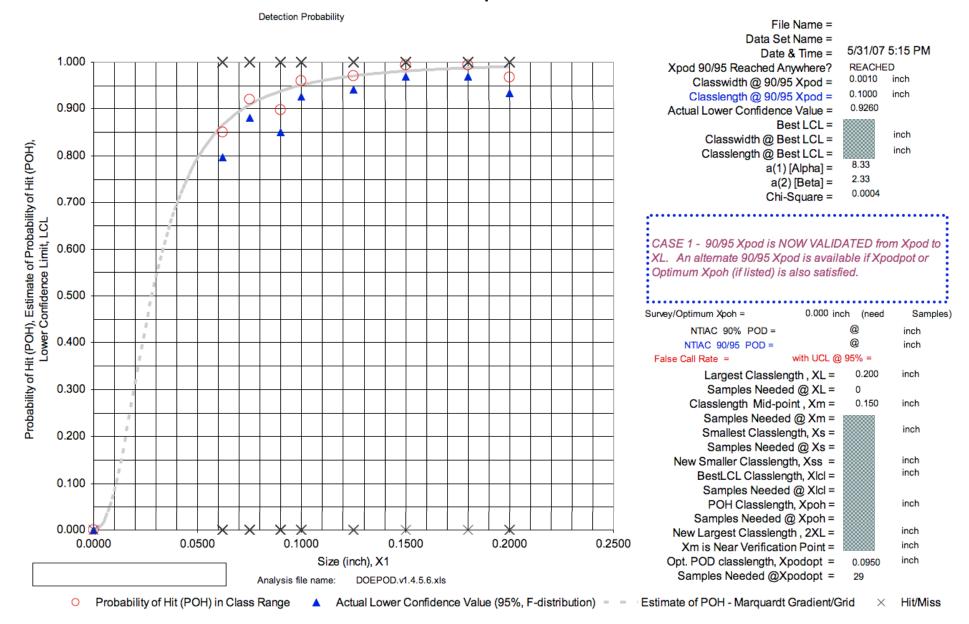
Optimum Xpoh (Best Xpod that could be achieved if test

Multi-			specimens are added)	DOEPOD	
Fasteners	Inspector	90/95 Xpod (in)	(in)	v1.4.5.6	5-Jun-07
	1	0.100	0.062	CASE 1	90/95 Xpod reached
	2	0.100	0.062	CASE 1	90/95 Xpod reached
	3	0.100	0.062	CASE 1	90/95 Xpod reached
	4	0.100	0.062	CASE 1	90/95 Xpod reached
	5	0.100	0.062	CASE 1	90/95 Xpod reached
	6	0.100	0.062	CASE 1	90/95 Xpod reached
	7	0.100	0.062	CASE 1	90/95 Xpod reached
	8	0.100	0.062	CASE 1	90/95 Xpod reached
	9	0.100	0.062	CASE 1	90/95 Xpod reached
	10	0.150	0.090	CASE 1	90/95 Xpod reached
	11	0.200	0.100	CASE 1	90/95 Xpod reached
	12	0.200	0.100	CASE 1	90/95 Xpod reached
	13	0.200	0.100	CASE 1	90/95 Xpod reached
	14	0.200	0.100	CASE 1	90/95 Xpod reached
	15	0.200	0.100	CASE 1	90/95 Xpod reached
	16	0.200	0.100	CASE 1	90/95 Xpod reached
	17	0.200	0.100	CASE 1	90/95 Xpod reached
	18	0.200	0.125	CASE 1	90/95 Xpod reached
	19	0.200	0.125	CASE 1	90/95 Xpod reached
					90/95 Xpod reached but
	20	0.100	see table A	CASE 2	miss at largest flaw size
	21	not reached		CASE 7	Miss at largest flaw size
	22	not reached		CASE 7	Miss at largest flaw size



DOEPOD Example (continued)

All Inspectors





False Calls

• False Calls are handled similarly except the upper confidence limit* P_u is used

$$False\ Call\ Rate = \frac{Number\ of\ False\ Calls\ (X)}{Number\ of\ False\ Call\ Opportunities\ (N)}$$

$$P_{u} = \frac{(X+1) F_{\alpha}(f_{1}, f_{2})}{(N-X) + (X+1) F_{\alpha}(f_{1}, f_{2})} , \begin{cases} f_{1} = 2(X+1) \\ f_{2} = 2(N-X) \end{cases}$$

• 95% Chance that the false call rate is less than or equal to the $\,P_{\scriptscriptstyle u}$



False Calls

- Test samples with no flaws present may be included in DOEPOD for determination of false call rate and the upper confidence value of the false call rate at 95% confidence. For test samples with no flaw present, enter flaw size of 0.00001"
- False call rate may be explored and optimized by adjusting signal amplitude threshold.
- Warning: May reach $90/95 X_{POD}$ at cost of increasing false call rate. Need to know what false call rate is acceptable.
- False calls rate should not be accepted as is without first addressing the cause of the false call and identifying procedures to remove false calls. May need to modify or add inspection protocols.



DOEPOD Data Entry

Hit / Miss Data

ID Number	CRACK SIZE (inches)	DEPTH	HIT/MISS (0 or 100)	Signal Amplitude Measured (Arbitrary Units)	SIGNAL TREASHOLD
	0.342		100		
	0.251		100		
	0.242		100		
	0.213		100		
	0.208		100		
	0.199		100		
	0.186		100		
	0.184		0		
	0.169		100		
	0.166		0		

Signal Amplitude Data

ID Number	CRACK SIZE (inches)	DEPTH	HIT/MISS (0 or 100)	Signal Amplitude Measured (Arbitrary Units)	SIGNAL TREASHOLD
	0.342			0.5	0.025
	0.251			0.4	
	0.242			0.3	
	0.213			0.2	
	0.208			0.1	
	0.199			0.05	
	0.186			0.025	
	0.184			0.01	
	0.169			0.025	
	0.166			0.01	

DOEPOD REQUIREMENTS



- Inspection processes are to be under control and fixed.
- Multiple inspection processes may be used on the same set of test samples with the constraint that Directed DOE POD is to be executed for each process separately. When multiple inspection processes or systems are used, the resulting directed sample requirements may be overlapping. In this situation, the user is to keep the non-overlapping directed sample requirements applied to the appropriate inspection process, while utilizing overlapping directed sample requirements for the multiple processes in order to minimize the number of generated test samples.
- There are to be an equal number of unflawed samples during any test.
- There are to be more than two (2) samples at different class lengths.
- A moving class width that groups flaws of similar size is used to optimize the lower confidence value. This moving class width and the best lower confidence bound (value) optimization will be invoked if there are more than four (4) samples at different class lengths.
- Flaw sizes must be greater than 0.00001"
- Test samples with no flaws present may be included for determination of false call rate and the upper confidence value of the false call rate at 95% confidence. For test samples with no flaw present, enter flaw size of 0.00001"
- The maximum number of test samples is 1999.
- Be prepared to generate, inspect, and evaluate test samples during the NDE technology capability determination.
- Validated 90/95 X_{POD} is obtained when the user has reached and satisfied the sample requirements of Case 1. That is, there is a 95% chance that the probability of detection of the system is greater than 90% for class lengths in the range 90/95 X_{POD} to XL.



SUMMARY

- Concept for Binomialization of Test Data
- Process for determining observed probability of hit (POH) and associated confidence limits
- Utilization of moving class width to group flaws and for flaw class width optimization
- Identification of POD CASES and directed actions needed to validate inspection systems.
- False call rate and confidence
- DOEPOD Data Entry
- DOEPOD Beta (2,500 lines of code, PC and MAC, limited distribution)
- Future work: DOEPOD upgrades
 - Interface with predicted POD MIL-HDBK-1823companion tool
 - ➤ Address very limited data sets when 90/95 X_{POD} can never be reached, and communicating those risks.

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- DOEPOD Probability of Hit estimating curve is for visualization only and not used in the DOEPOD analysis.
- Probability of Hit estimating curve is not to be used for validation or for justification of validation.
- The default function used in DOEPOD is:

$$POH = Exp(a(1) + a(2) * Log(x1)) / (1 + Exp(a(1) + a(2) * Log(x1)))$$
 $a(1) = \alpha$
 $a(2) = \beta$
 $x1 = Flaw size$

Other multi-parameter functions may be used